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Variable Fluid Properties Effects on Hydromagnetic Fluid Flow over an Exponentially Stretching Sheet

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Abstract

In this work, the problem of heat and mass transfer by laminar flow of Newtonian, viscous, electrically conducting fluid past an exponentially stretching permeable sheet with variable heat and mass fluxes in the presence of non-uniform magnetic field is studied The effects of non-uniform heat generation/absorption and thermal radiation are included in the boundary layer equations. Using similarity transformations, the partial differential equations governing the flow are transformed into a system of coupled nonlinear ordinary differential equations which is solved numerically by fourth-order Runge–Kutta method using the shooting technique. The effects of various pertinent parameters on the local skin- friction coefficient, the local Nusselt number and the local Sherwood number are explained graphically and discussed.

Keywords

Exponentially Stretching Sheet, Variable Fluid Properties, Non-Uniform Heat Generation, Thermal Radiation

1. Introduction

The study of heat and mass transfer over a stretching surface is an important type of flow occurring in many manufacturing processes. For examples, hot rolling, aerodynamic extrusion of plastic sheet, wire drawing, glass-fiber, the cooling of metallic plate in a bath, etc. Crane [1] was the first one who investigated the flow caused by a linearly stretching sheet. Many authors [2-10] extended the work of Crane under various situations.

In the above studies, the researchers deal with linear variation of stretching velocity of the sheet. In recent years, the problem of the boundary layer flow over an exponentially stretching sheet has considerable attention due to its applications in many engineering processes. The boundary layer flow, heat and mass transfer over an exponentially stretching sheet investigated by Magyari and Keller [11]. Mandal and Mukhopadhyay [12] numerically examined the flow and heat transfer over an exponentially stretching porous sheet embedded in a porous medium with variable heat flux. Ahmad et al. [13] investigated the effects of thermal radiation and variable thermal conductivity on MHD boundary layer flow and heat transfer of a viscous fluid past an exponentially stretching sheet immersed in a porous medium. Rahman et al. [14] presented analytical solutions, using homotopy analysis method, and numerical solutions, using Killer box method, for the problem of the mixed convection stagnation point flow of a third grade fluid over an exponentially stretching sheet. Mukhopadhyay et al. [15] examined the flow and mass transfer of a viscous fluid with solute distribution in the fluid towards an exponentially stretching porous sheet in a stratified medium in the presence of first order chemical reaction. Mukherjee and Parsad [16] studied the effect of thermal radiation and uniform magnetic field on the boundary layer flow and heat transfer of Newtonian fluid over an exponentially stretching sheet embedded in a porous medium in an exponential free stream. Mabood et al. [17] analytically examined the effects of radiation on boundary layer flow of a viscous Newtonian fluid over an exponentially stretching sheet using homotopy perturbation method.

Unfortunately, the local skin-friction coefficient and the surface heat flux were not correctly derived in the papers by Megahed [18-22], Liu and Megahed [23], Khader and Megahed [24], Dimian and Megahed [25], Liu and Megahed [26] and Liu et al.[27] so that the results of these papers are inaccurate. Therefore, the objective of this paper is to improve and extend the work of Megahed [18] by considering the effect of magnetic field and variable thermal conductivity on a Newtonian viscous fluid flow, heat and mass transfer characteristics over an exponentially stretching permeable sheet with variable surface heat and mass fluxes.

2. Mathematical Formulation

Consider the steady boundary layer flow of an electrically conducting viscous fluid over an exponentially permeable stretching sheet coinciding with the plane $y = 0$, the fluid fills the porous space above the surface $y > 0$. Two equal and opposite forces are introduced along the x-axis, so that the surface is stretched keeping the origin fixed and y-axis perpendicular to it. A non-uniform magnetic field $B(x)=\frac{B_0}{\sqrt{2}}e^{x/2L}$ is applied normally to the sheet .

It is assumed that the magnetic Reynolds number is small so that the induced magnetic field is neglected. In addition no applied electric field and the Hall effect, viscous dissipation and Joule heating are neglected. With these assumptions and using the boundary layer approximations, the continuity, momentum, energy and mass diffusion equations in the presence of heat generation/absorption and thermal radiation are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y})-\frac{\sigma B^2(x)}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}(\kappa\frac{\partial T}{\partial y})+\frac{q'''}{\rho c_p}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}. \tag{4}$$

By using Rosseland approximation for radiation[28] we have:

$$q_r=-\frac{4\sigma^*}{\kappa^*}\frac{\partial T^4}{\partial y}, \tag{5}$$

where σ^* is the Stefan-Boltzmann constant and κ^* is the mean absorption coefficient. Assuming that the temperature differences within the flow are such that T^4 may be expressed as a linear function of the temperature. This can be obtained by expanding T^4 in a Taylor series about the ambient temperature T_∞ and neglecting higher order terms, then:

$$T^4 \cong 4T_\infty^3 T-3T_\infty^4. \tag{6}$$

The Fourier's law of conduction in the presence of thermal radiation is:

$$q_{eff}=-(\kappa+\frac{16\sigma^* T_\infty^3}{3\kappa^*})\frac{\partial T}{\partial y}=-\kappa_{eff}\frac{\partial T}{\partial y}, \tag{7}$$

where $\kappa_{eff}=\kappa+\frac{16\sigma^* T_\infty^3}{3\kappa^*}$ is the effective thermal conductivity.

The appropriate boundary conditions are:

$$u=u_w(x),\quad v=-v_w(x),\quad -\kappa_{eff}\frac{\partial T}{\partial y}=Ae^{\frac{(a+1)x}{2L}},$$

$$-D\frac{\partial C}{\partial y}=B\,e^{\frac{(b+1)x}{2L}}\quad at\quad y=0,$$

$$u\to 0,\quad T\to T_\infty,\quad C\to C_\infty\quad as\quad y\to\infty. \tag{8}$$

Here u and v are the velocity components of the velocity in the $x-$ and $y-$ directions, respectively. T is the temperature, C is the concentration, μ is the fluid viscosity, ρ is the density, σ is the electrical conductivity, κ is the thermal conductivity, c_p is the specific heat at constant pressure, q_r is the radiative heat flux, D is the mass diffusion, $u_w(x)=u_0e^{x/L}$ is the stretching sheet velocity, $L>0$ is the characteristic length, u_0 is the characteristic velocity, A, B, a and b are constants. T_∞ is the ambient temperature and C_∞ is the ambient concentration. $v_w(x)=v_0e^{x/2L}$ is the normal wall velocity, where v_0 is a constant. $v_0>0$ is the velocity of suction and $v_0<0$ is the velocity of blowing.

Introducing the following dimensionless variables:

$$\eta=\sqrt{\frac{\rho u_0}{2\mu_\infty L}}\,y\,e^{x/2L},\quad u(x,y)=u_0\,e^{x/L}f'(\eta),$$

$$v(x,y)=-\frac{\mu_\infty}{\rho L}\sqrt{\frac{\mathrm{Re}}{2}}e^{x/2L}(f(\eta)+\eta f'(\eta)), \tag{9}$$

$$\theta=\frac{T-T_\infty}{\sqrt{\frac{2\mu_\infty L}{\rho u_0}}\frac{A}{\kappa_\infty}e^{ax/2L}},\quad \phi=\frac{C-C_\infty}{\sqrt{\frac{2\mu_\infty L}{\rho u_0}}\frac{B}{D}e^{bx/2L}},$$

where $\mathrm{Re}=\frac{Lu_0\rho}{\mu_\infty}$ is the Reynolds number, θ is the dimensionless temperature, ϕ is the dimensionless concentration and q''' is the non-uniform heat source/sink given as (see Bataller [29] and Mahmoud [30]):

$$q'''=\frac{\rho\kappa_\infty u_0}{2L\mu_\infty}e^{x/L}(\frac{A}{\kappa_\infty}\sqrt{\frac{2\mu_\infty L}{\rho u_0}}a^*e^{-\eta}+b^*(T-T_\infty)), \tag{10}$$

where a^* and b^* are the coefficient of space and temperature dependant heat generation/absorption, respectively. Note that $a^*>0, b^*>0$ corresponds to internal heat generation, while $a^*<0, b^*<0$ corresponds to internal heat absorption.

In the present work, the fluid viscosity is assumed to vary as an exponential function of temperature in the dimensionless form as (see Mahmoud [31] and Nadeem and Awais[32]):

$$\mu = \mu_\infty e^{-\alpha\theta}, \tag{11}$$

the temperature dependant thermal conductivity defined as (see Abel and Mhesha [33], Mahmoud [34], Mahmoud and Waheed [35] and El-Hawary et al.[36]):

$$\kappa = \kappa_\infty (1 + \beta\theta), \tag{12}$$

where μ_∞ and κ_∞ are the fluid viscosity and thermal conductivity at temperature T_∞. α is the viscosity parameter and β is the thermal conductivity parameter. α and β depend on the nature of the fluid.

In view of Eqs. (9)-(12), Eqs. (1)-(4) reduced to:

$$e^{-\alpha\theta}(f''' - \alpha\theta' f'') + f f'' - 2f'^2 - M f' = 0, \tag{13}$$

$$(1 + R + \beta\theta)\theta'' + \beta\theta'^2 + (a^* e^{\eta} + b^* \theta) + \Pr(f\theta' - a\theta f') = 0, \tag{14}$$

$$\phi'' + Sc(f\phi' - b\phi f') = 0. \tag{15}$$

The boundary conditions (8) become:

$$f' = 1, \quad f = f_w, \quad \theta' = -\frac{1}{(1+\beta\theta+R)}, \quad \phi' = -1, \quad at\ \eta = 0, \tag{16}$$

$$f' \to 0, \qquad \theta \to 0, \quad \phi \to 0, \quad as\ \eta \to \infty .$$

Where $f_w = -v_0\sqrt{\dfrac{2\rho L}{u_0\,\mu_\infty}}$ is the suction or blowing parameter, where physically $f_w > 0$ corresponds to suction at the sheet and $f_w < 0$ corresponds to blowing at the sheet. $Sc = \dfrac{\mu_\infty}{\rho D}$ is the Schmidt number, $\Pr = \dfrac{\mu_\infty c_p}{k_\infty}$ is the Prandtl number, $R = \dfrac{16\sigma^* T_\infty^3}{3\kappa^* \kappa_\infty}$ is the thermal radiation parameter.

Important physical parameters of interest in this problem are the local skin- friction Cf_x, the local Nusselt number Nu_x and the local Sherwood number Sh_x which are respectively defined as:

$$Cf_x = -(\frac{\mathrm{Re}_x}{2})^{-1/2} e^{-\alpha\theta(0)} f''(0)\,, \tag{17}$$

$$Nu_x = (\frac{x}{2L}\mathrm{Re}_x)^{1/2}\frac{1}{\theta(0)}, \tag{18}$$

$$Sh_x = (\frac{x}{2L}\mathrm{Re}_x)^{1/2}\frac{1}{\phi(0)}, \tag{19}$$

where $\mathrm{Re}_x = \dfrac{\rho u_0 e^{x/L} x}{\mu_\infty}$ is the local Reynolds number.

3. Numerical Results and Discussion

Eqs. (13)-(15) with boundary conditions (16) were solved numerically using the fourth order Runge - Kutta integration scheme with the shooting method. Figs. 1-8 exhibit the local skin- friction, the local Nusselt number and the local Sherwood number assigning numerical values to the magnetic parameter M, the viscosity parameter α, the thermal conductivity parameter β, the radiation parameter R, suction /injection parameter f_w and the heat generation/ absorption parameters a^*, b^*. It is seen that the values of $f''(0)$ are always negative. Physically, negative sign of $f''(0)$ implies that the stretching sheet exerts a drag force on the fluid that cause the movement of the fluid on the surface.

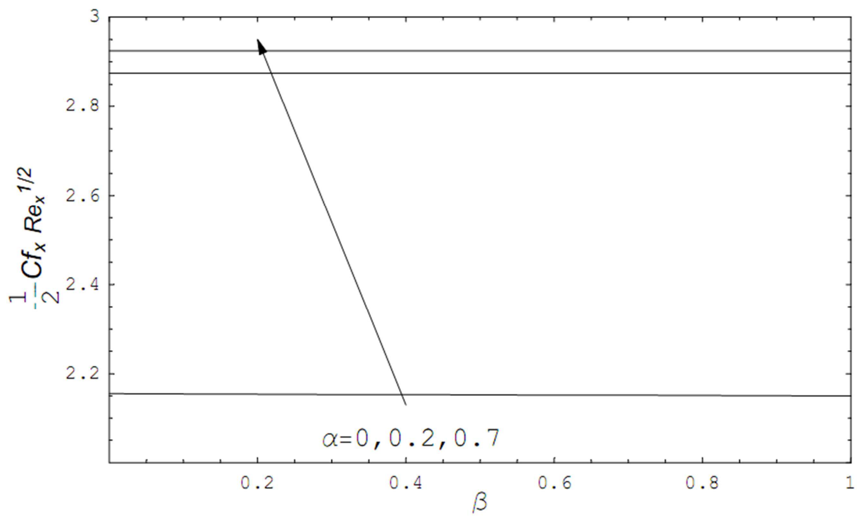

Fig. 1. *Variation of the local skin friction coefficient as a function of* β *for various values of* α *with* $M = 0.3, f_w = 1.5, R = 0.4, a = b = -1, a^* = b^* = -0.2, \Pr = 1$ *and* $Sc = 0.6$.

Fig. 1 shows the local skin- friction coefficient in the boundary layer for various values of the parameters α and β . This figure depicts that, increase in the value of the parameter α leads to an increase in the local skin- friction coefficient . Also,It is shown that the thermal conductivity parameter β has no significant effect on local skin- friction coefficient .

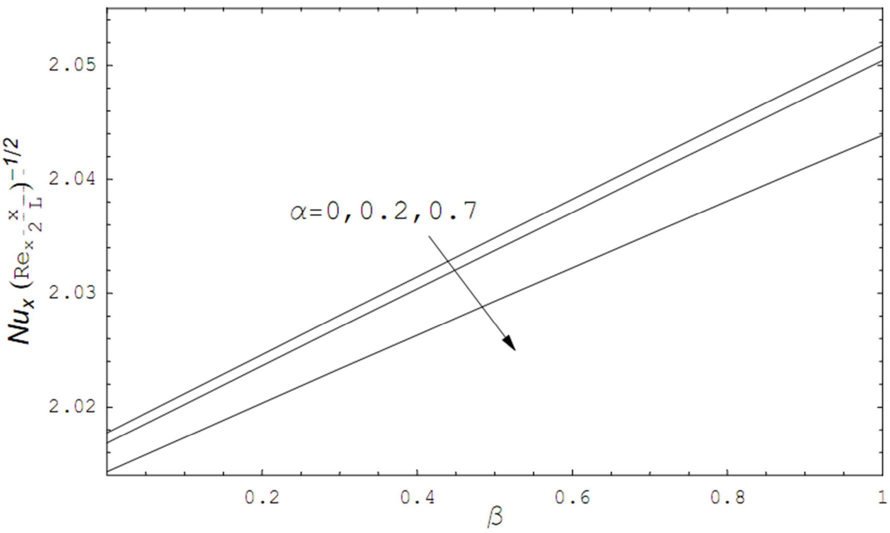

Fig. 2. *Variation of the local Nusselt number as a function of* β *for various values of* α *with* $M = 0.3,\ f_w = 1.5,\ R = 0.4,\ a = b = -1,\quad a^* = b^* = -0.2,\ \Pr = 1$ *and* $Sc = 0.6.$

Fig. 2 shows the local Nusselt number for various values of the parameters α and β . This figure depicts that, increase in the value of the parameter α leads to decrease in the local Nusselt number within the boundary layer. Whereas, the influence of increasing the thermal conductivity parameter β is increasing the local Nusselt number .

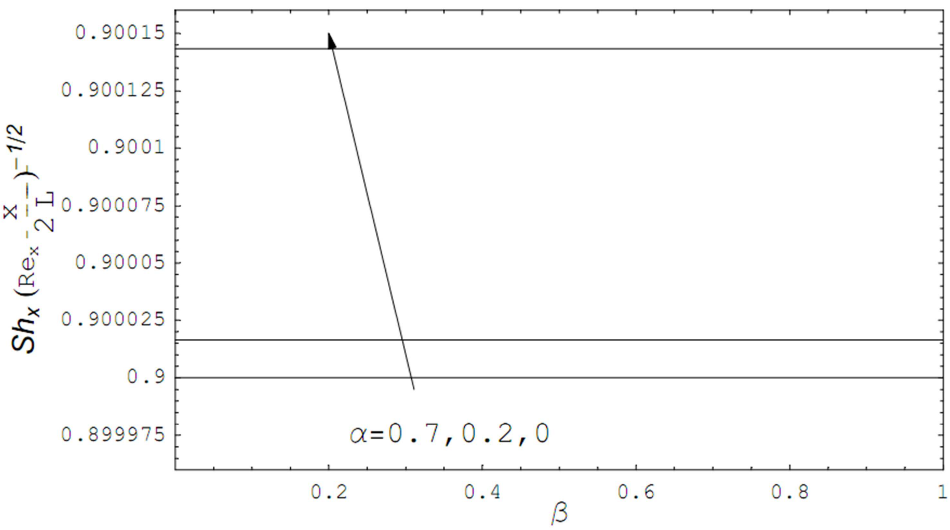

Fig. 3. *Variation of the local Sherwood number as a function of* β *for various values of* α *with* $M = 0.3,\ f_w = 1.5,\ R = 0.4,\ a = b = -1,\quad a^* = b^* = -0.2,\ \Pr = 1$ *and* $Sc = 0.6.$

Fig. 3 shows the local Sherwood number for various values of the parameters α and β . It can be seen that, increase in the value of the parameter α leads to decrease in the local Sherwood number within the boundary layer. Whereas, the increasing of the thermal conductivity parameter β has no significant effect on the local Sherwood number.

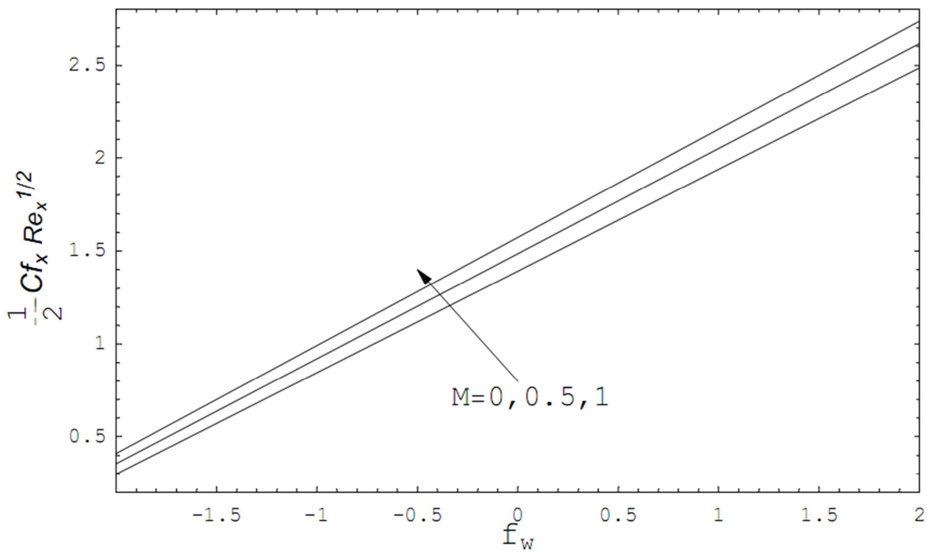

Fig. 4. *Variation of the local skin friction coefficient as a function of* f_w *for various values of* M *with* $\alpha = 0.5, \beta = 0.1, R = 0.5, \Pr = 1, a = b = 1/3, a^* = b^* = 0.2$ *and* $Sc = 0.6$.

The effects of the suction/injection parameter f_w and the magnetic parameter M on the local skin-friction coefficient is displayed in Fig. 4. It is evident that the local skin-friction coefficient increases with the increase of the magnetic parameter. Physically, these behaviors due to the fact that application of a magnetic field to an electrically conducting fluid produced a drag-like force as known Lorentz force. This force causes reduction in the fluid velocity and hence an increasing in local skin-friction coefficient. Actually, the suction parameter $(f_w > 0)$ increases the local skin- friction coefficient . While, the absolute value of the injection parameter $(f_w < 0)$ has the effect of decreasing the local skin- friction coefficient.

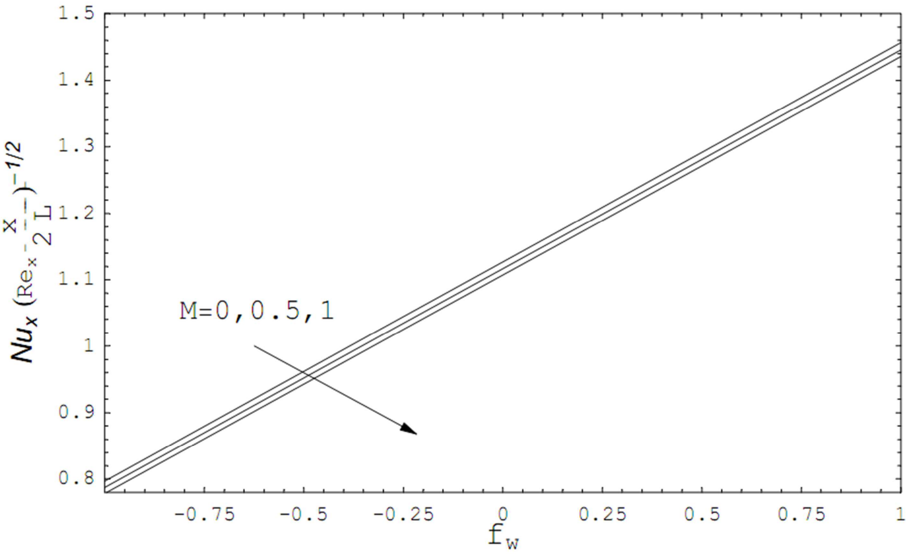

Fig. 5. *Variation of the local Nusselt number as a function of* f_w *for various values of* M *with* $\alpha = 0.5, \beta = 0.1, R = 0.5, \Pr = 1, a = b = 1/3, a^* = b^* = 0.2$ *and* $Sc = 0.6$.

The effects of the suction/injection parameter f_w and the magnetic parameter M on the local Nusselt number is displayed in Figs. 5. It is noticed that the local Nusselt number decreases with the increases of M .Also,its found that increasing the suction parameter $(f_w > 0)$ leads to an increase the local Nusselt number. While, the absolute value of the injection parameter $(f_w < 0)$ has the effect of decreasing the local Nusselt number.

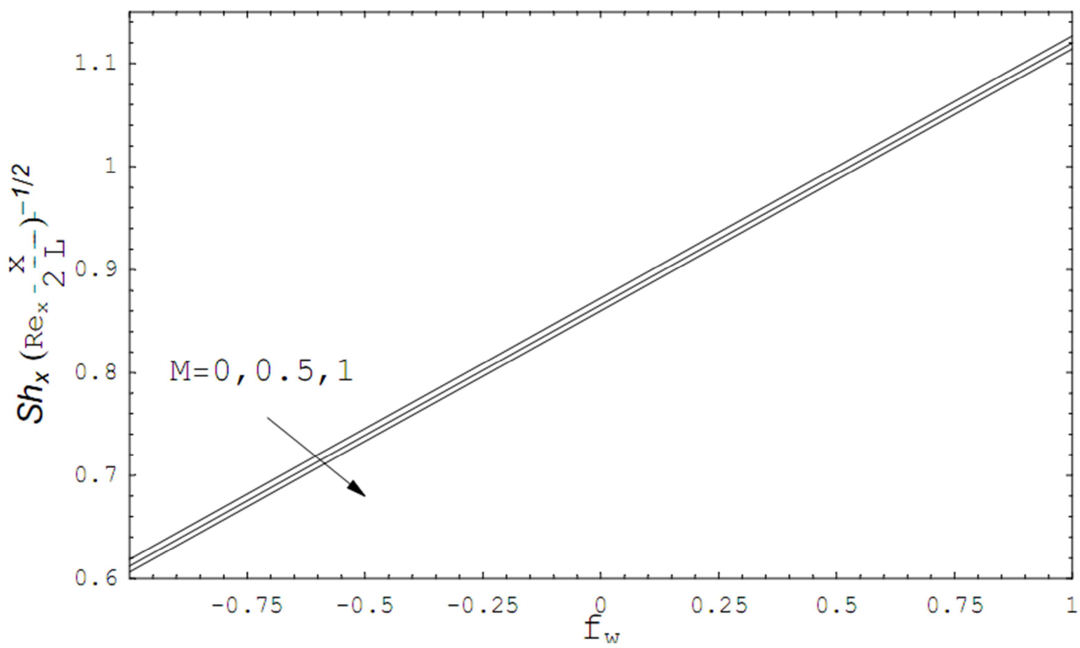

Fig. 6. *Variation of the local Sherwood number as a function of* f_w *for various values of* M *with* $\alpha = 0.5$, $\beta = 0.1$, $R = 0.5$, $\Pr = 1$, $a = b = 1/3$, $a^* = b^* = 0.2$ *and* $Sc = 0.6$.

The effects of the suction/injection parameter f_w and the magnetic parameter M on the local Sherwood number is displayed in Fig. 6. It is observed that the local Sherwood number decreases with the increases of M . It is also found that as the suction parameter ($f_w > 0$) increases the local Sherwood number. While, the absolute value of the injection parameter ($f_w < 0$) has the effect of decreasing the local Sherwood number .

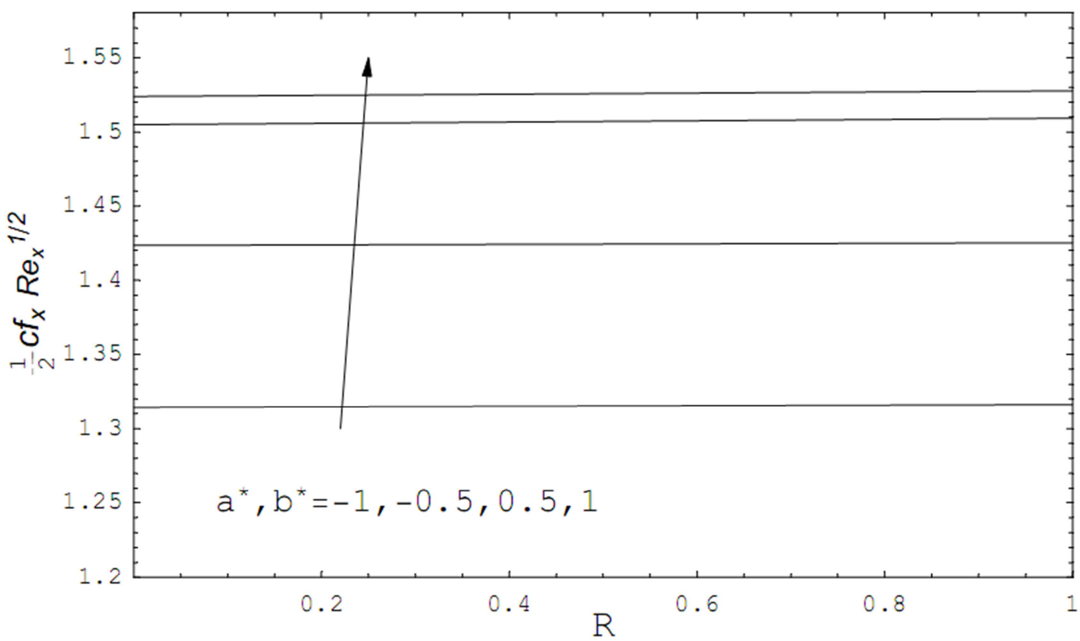

Fig. 7. *Variation of the local skin friction coefficient as a function of* R *for various values of* a^*, b^* *with* $M = 0.1$, $f_w = -0.1$, $R = 0.5$, $a = b = 1/2$, $\alpha = 0.5$, $\beta = 0.1$, $\Pr = 1$ *and* $Sc = 0.6$.

Fig. 7 illustrates the variation of the local skin- friction coefficient with the thermal radiation R for different values of the non-uniform heat generation/absorption parameters a^*, b^* . It is shown that the radiation parameter has no significant effect on the local skin –friction coefficient. It is also found that the heat generation parameters ($a^*, b^* > 0$) have the effect of increasing the local skin- friction coefficient. But, the local skin- friction coefficient decreases as the absolute value of the heat absorption parameters ($a^*, b^* < 0$) increase.

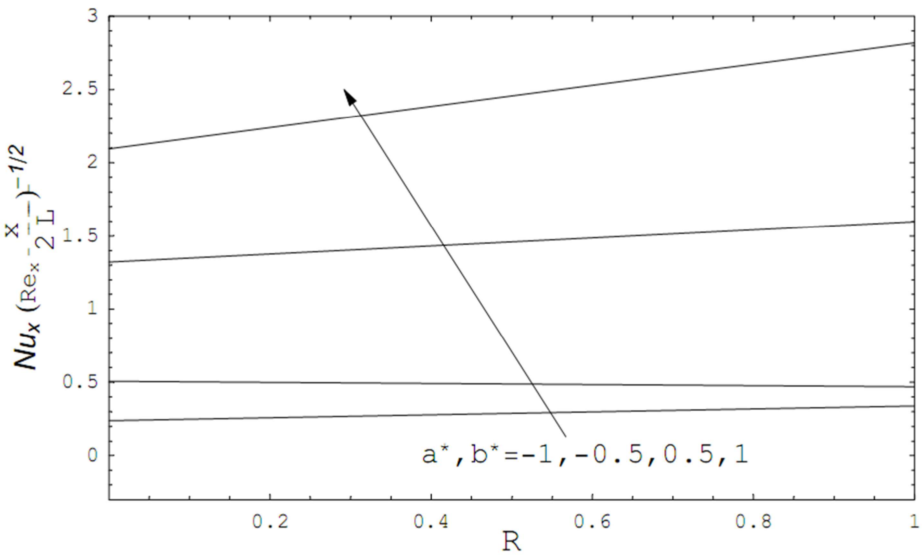

Fig. 8. *Variation of the local Nusselt number as a function of* R *for various values of* a^*, b^* *with* $M = 0.1, f_w = -0.1, R = 0.5, \quad a = b = 1/2, \alpha = 0.5, \beta = 0.1, \Pr = 1$ *and* $Sc = 0.6$.

Fig. 8 shows the variation of the local Nusselt number Nu_x with the thermal radiation R for different values of the non-uniform heat generation/absorption parameters a^*, b^*. It is noticed that an increasing in the radiation parameter leads to an increasing in the local Nusselt number where ($a^*, b^* > 0$) and decreasing where ($a^*, b^* > 0$). It is observed that the heat generation parameters ($a^*, b^* > 0$) have the effect of increasing the local Nusselt number. While the local Nusselt number decreases as the absolute value of the heat absorption parameters ($a^*, b^* < 0$) increase.

4. Conclusions

In this study, the effects of radiation and heat generation/absorption on MHD heat and mass transfer of a Newtonian fluid over an exponentially stretching surface with variable heat and mass fluxes in the presence of heat suction/injection were discussed. The results indicate that the variable viscosity parameter, magnetic parameter, the suction parameter and the heat generation parameter enhancing the local skin-friction coefficient. The local Nusselt number increased with the increase of the suction parameter, radiation parameter, the suction parameter and the heat generation parameter. Also, the local Sherwood number increased with the increase of the suction parameter.

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